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First Experiment to Test Whether Space-Time Is Flat

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From both special and general relativistic gravitational theories, we deduce the predicted value of the precessional angular velocity of a gyroscope caused by the additional gravitational field due to the earth's rotation. The orbiting gyroscope experiment will be the first to test whether space-time is flat.

In previous papers (Zhang Junhao and Chen Xiang, 1990, 1991) we set up the special relativistic gravitational theory and compared it with general relativity. In these theories, gravity may be expressed as

$$\mathbf{F}^{(S)} = m\left(\mathbf{E}^{(S)} + \frac{1}{c}\mathbf{u}\times\mathbf{B}^{(S)} + \frac{1}{c}\mathbf{u}\cdot\mathscr{P}^{(S)} + \frac{1}{c^2}\mathbf{u}\times\mathscr{R}\cdot\mathbf{u}\right) \tag{1}$$

$$\mathbf{F}^{(G)} = m\left(\mathbf{E}^{(G)} + \frac{1}{c}\mathbf{u}\times\mathbf{B}^{(G)} + \frac{1}{c}\mathbf{u}\cdot\mathscr{P}^{(G)} + \frac{1}{c^2}\mathbf{u}\times\mathscr{R}\cdot\mathbf{u}\right) + \mathbf{D}^{(G)} \tag{2}$$

The theoretical values of the redshift, the angle of deflection of light, and the planetary perihelion shift only relate to the E_i and R_{ij} components of the field. Since the E_i and R_{ij} components of both theories are the same, we cannot determine which gravitational theory is correct and cannot judge whether space-time is curved from these observational values.

The two gravitational theories predict different values of the B_i and P_{ij} components of the field due to the earth's rotation:

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$$B_i^{(S)}=\frac{-3GM\Omega R_0^2}{5cR^5}(3x_1x_3,\,3x_2x_3,\,-x_1^2-x_2^2+2x_3^2)$$

$$P_{ij}^{(S)}=\frac{3GM\Omega R_0^2}{5cR^5}\begin{pmatrix}-2x_1x_2 & x_1^2-x_2^2 & -x_2x_3\\ x_1^2-x_2^2 & 2x_1x_2 & x_1x_3\\ -x_2x_3 & x_1x_3 & 0\end{pmatrix} \tag{3}$$

$$B_i^{(G)}=\tfrac{4}{3}B_i^{(S)}$$

$$P_{ij}^{(G)}=0$$

where M, R_0, and Ω are the mass, radius, and angular velocity of the earth, respectively. By measuring the values of B_i and P_{ij}, we may determine which gravitational theory is correct. Stanford University is planning to set four rotating gyroscopes in an orbiting satellite and measure the precessional angular velocity of the gyroscopes as the satellite revolves around the earth. It is the first experiment to judge whether space-time is flat.

Suppose $\boldsymbol{\omega}$ is the angular velocity of the gyroscope; the element of the moment of gravity exerted on an element volume dv of the gyroscope is

$$d\mathbf{M}=\frac{1}{c}\,\mathbf{r}\times[(\boldsymbol{\omega}\times\mathbf{r})\times\mathbf{B}+(\boldsymbol{\omega}\times\mathbf{r})\cdot\mathscr{P}]\rho\,dv \tag{4}$$

If the gyroscope is a spherical body, the total moment of gravity is

$$\mathbf{M}=\int d\mathbf{M}=\frac{I}{2c}(\boldsymbol{\omega}\times\mathbf{B}-\boldsymbol{\omega}\cdot\mathscr{P}) \tag{5}$$

where $I=2mr_0^2/5$. Suppose that T is the period of revolution of the satellite; the average moment of gravity is

$$\mathbf{M}_{av}=\frac{1}{T}\int_0^T\mathbf{M}\,dt=\frac{I}{2c}\left[\boldsymbol{\omega}\times\left(\frac{1}{T}\int_0^T\mathbf{B}\,dt\right)-\boldsymbol{\omega}\cdot\left(\frac{1}{T}\int_0^T\mathscr{P}\,dt\right)\right]$$

$$=\frac{I}{2c}(\boldsymbol{\omega}\times\mathbf{B}_{av}-\boldsymbol{\omega}\cdot\mathscr{P}_{av}) \tag{6}$$

Let ox_3 be the earth's axis, $o\xi$ the normal line of the orbit plane of the satellite, and ox_1 the line in which the $ox_3\xi$ plane intersects the equatorial plane; we have

$$(B_{\rm av})_i = A(-3\cos\theta\sin\theta, 0, 3\cos^2\theta - 1)$$

$$(p_{\rm av})_{ij} = A\begin{pmatrix} 0 & -\sin^2\theta & 0 \\ -\sin^2\theta & 0 & \cos\theta\sin\theta \\ 0 & \cos\theta\sin\theta & 0 \end{pmatrix} \tag{7}$$

$$A = \frac{3GMR_0^2\Omega}{10a^3(1-e^2)^{3/2}c}$$

in the $ox_1x_2x_3$ frame system, where θ is the angle between ox_3 and $o\xi$, and a and e are the major radius and eccentricity of the satellite's orbit, respectively. We omit the index "av" in the following discussion. Then we have

$$\frac{d\omega_1}{dt} = \frac{A}{c}\cos^2\theta\,\omega_2$$

$$\frac{d\omega_2}{dt} = \frac{A}{c}[(1-2\cos^2\theta)\omega_1 - 2\cos\theta\sin\theta\,\omega_3] \tag{8}$$

$$\frac{d\omega_3}{dt} = \frac{A}{c}\sin\theta\cos\theta\,\omega_2$$

Let

$$\mathbf{e}_\zeta = \cos\theta\,\mathbf{e}_1 + \sin\theta\,\mathbf{e}_3, \qquad \mathbf{e}_\eta = \mathbf{e}_2, \qquad \mathbf{e}_\xi = -\sin\theta\,\mathbf{e}_1 + \cos\theta\,\mathbf{e}_3$$

Then we have

$$\frac{d\omega_\zeta}{dt} = \frac{A}{c}\cos\theta\,\omega_\eta$$

$$\frac{d\omega_\eta}{dt} = \frac{-A}{c}(\cos\theta\,\omega_\zeta + \sin\theta\,\omega_\xi) \tag{9}$$

$$\frac{d\omega_\xi}{dt} = 0$$

and

$$\omega_\xi = \text{const}, \qquad \omega_\eta^2 + (\omega_\zeta + \tan\theta\,\omega_\xi)^2 = \text{const} \tag{10}$$

Set the origin of $\boldsymbol{\omega}$ at o; the end of $\boldsymbol{\omega}$ is in eccentric circular motion in the plane perpendicular to $\mathbf{e}_\xi$; the magnitude of $\boldsymbol{\omega}$ varies periodically. If $\mathbf{e}_\omega$ is the unit vector of $\boldsymbol{\omega}$, we have

$$\frac{d\mathbf{e}_\omega}{dt}=\frac{1}{\omega}\frac{d\boldsymbol{\omega}}{dt}-\frac{1}{\omega^3}\left(\frac{d\boldsymbol{\omega}}{dt}\cdot\boldsymbol{\omega}\right)\boldsymbol{\omega} \tag{11}$$

The precessional angular velocity of the gyroscope is

$$\begin{aligned}\dot{\psi}&=\left|\frac{d\mathbf{e}_\omega}{dt}\right|\\ &=\frac{A}{c\omega^2}[\omega^2(\omega_\zeta^2+\omega_\eta^2)\cos^2\theta+2\omega^2\omega_\zeta\omega_\xi\cos\theta\sin\theta\\ &\quad+\omega_\xi^2(\omega_\zeta^2+\omega_\xi^2)\sin^2\theta]^{1/2}\end{aligned} \tag{12}$$

Let $\omega_\zeta=\omega\sin\alpha\cos\beta$, $\omega_\eta=\omega\sin\alpha\sin\beta$, and $\omega_\xi=\omega\cos\alpha$; we can prove that

$$\dot{\psi}(\alpha,\beta)<\dot{\psi}(\alpha,\beta=0),\qquad \text{or}\qquad \dot{\psi}(\alpha,\beta)<\dot{\psi}(\alpha,\beta=\pi) \tag{13}$$

In the $\omega_\eta=0$ case,

$$\dot{\psi}^{(S)}_{\omega_\eta=0}=\begin{cases}(A/c)|\sin\theta\cos\alpha+\cos\theta\sin\alpha| & \text{if}\quad \omega_\zeta>0\\ (A/c)|\sin\theta\cos\alpha-\cos\theta\sin\alpha| & \text{if}\quad \omega_\zeta<0\end{cases} \tag{14}$$

This is the predicted value of the special relativistic gravitational theory.

From (2) and (3) we have

$$\frac{d\boldsymbol{\omega}}{dt}=\frac{1}{2c}\boldsymbol{\omega}\times\mathbf{B}^{(G)} \tag{15}$$

in general relativity. The end of $\boldsymbol{\omega}$ is in circular motion in the plane perpendicular to $\mathbf{B}$. The magnitude of $\boldsymbol{\omega}$ is constant. Let the angle between $\mathbf{B}$ and $\boldsymbol{\omega}$ be δ. The predicted value of the precessional angular velocity of general relativity is

$$\dot{\psi}^{(G)}=\frac{1}{2c}B^{(G)}\sin\delta=\frac{2A}{3c}(1+3\cos^2\theta)^{1/2}\sin\delta$$

$$\sin\delta|_{\omega_\eta=0}=\begin{cases}(1+3\cos^2\theta)^{-1/2}|\sin\theta\cos\alpha+2\cos\theta\sin\alpha| & \text{if}\quad \omega_\zeta>0\\ (1+3\cos^2\theta)^{-1/2}|\sin\theta\cos\alpha-2\cos\theta\sin\alpha| & \text{if}\quad \omega_\zeta<0\end{cases} \tag{16}$$

If $\theta < \pi/4$ and $\omega_\zeta > 0$, we get

$$[\dot{\psi}^{(G)} - \dot{\psi}^{(S)}]|_{\omega_\eta = 0} = \frac{A}{3c}\sin(\alpha - \theta) \tag{17}$$

from (14) and (16). If $\alpha = \theta + \pi/2$, the difference of the two predicted values is maximum. In this case,

$$\begin{aligned} \dot{\psi}^{(S)}|_{\omega_\eta = 0} &= \frac{A}{c}(2\cos^2\theta - 1) \\ \dot{\psi}^{(G)}|_{\omega_\eta = 0} &= \frac{A}{c}(2\cos^2\theta - \tfrac{2}{3}) \end{aligned} \tag{18}$$

If $\theta = 0$ and $\omega_\xi = 0$, we have

$$\dot{\psi}^{(S)}/\dot{\psi}^{(G)} = \tfrac{3}{4} \tag{19}$$

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